Numerical Simulation of Shear Resisting Mechanism and Shear Strength Equation for Box and Open Sandwich Beams

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Introduction

With the advancement of innovative design of composite structures for civil engineering construction, steel-concrete sandwich structures (Fig. 1) have added a new dimension for the contentment of structural designer, when construction restrictions and safely become a vital issue than construction cost. Steel - concrete composite beam, more specifically, Open and Box Sandwich beams, is such a new solution to construct structures like bridge girder, submerged box tunnel etc. Most of these heavy duty structures sustain a substantially high amount of dead/live load than the traditional RC structures. Sometime sudden impulse load on a small area may cause shear failure which is so catastrophic in nature that it does not give sufficient warning prior to failure with the primary crushing of concrete as a consequence rather than yielding of flexural steel. In this case, it is crucial to visualize the shear resisting mechanism and eventually evaluate shear strength with good accuracy.

Unlikely to RC beams, normally sandwich beams are designed with a larger amount of steel in a fashion of skin plate and embedded web plate (Fig. 2 & Fig. 3). Without concrete most of the sandwich beams look like typical steel I-beam.

Shear compression failure of surrounded concrete is very common where shear span to depth ratio is no too high to lead a flexural failure. Due to the confinement of concrete by steel skin plate sandwich beam shows quite ductile failure behavior than explosive type of shear tension failure

The conventional method of shear design for RC member can not be applied directly to sandwich member due to the difference in steel reinforcement configuration and arrangement.

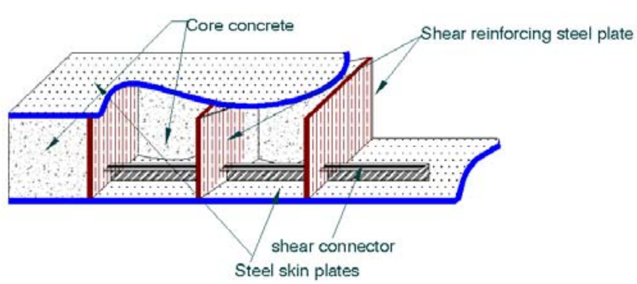

Fig. 1 A typical sandwich member

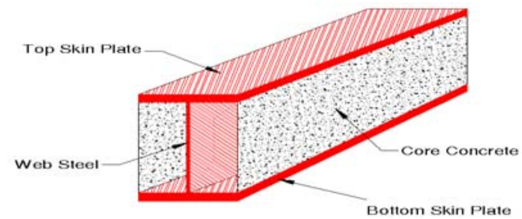

Fig. 2 A typical box sandwich beam.

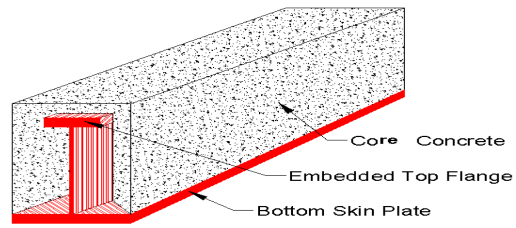

Fig. 3 A typical open sandwich beam

In sandwich beam, shear connector is provided between steel-concrete interface to curb the direct shear failure at this predetermined plane of weakness.

Approach to Predict Shear Strength

Considering the contemporary research on sandwich beam, there are two basic approaches,

- Conventional approach
- Numerical approach

Conventional Approach

Most of the past research to predict the shear strength of sandwich beam based on three popular analogies.

- Classical beam theory
- Truss analogy (compression field theory)
- Tied-arch mechanism

Among the above three, truss mechanism as shown in Fig. 4, is the simplest one where shear resistance by concrete in compression zone and aggregate interlocking force along the diagonal shear crack is totally ignored. Only the diagonal compression force is considered as concrete contribution to shear resistance. Truss analogy is adopted when a sandwich beam is designed with a stirrup type web reinforcement such as single, double or multiple tie plates or diaphragm web.

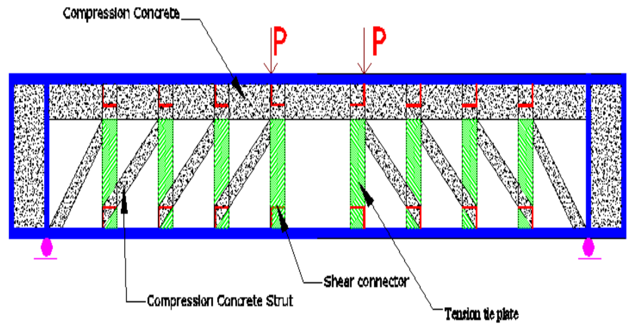

Fig. 4 Truss action in a box sandwich beam

Tied-arch mechanism is quite applicable for beam with a shear span to depth ratio is less than 2. Beam of this category commonly known as deep beam. Because of this proportions, they are likely to have strength controlled by shear and have a higher shear capacity over usual beams of a/d greater than 2.

The mechanism of shear resistance is quite different in deep beam from that of usual beam of normal proportion. After inclined crack occurs, this beam tends to behave like a tide arch wherein the load is carried by direct compression in concrete strut (Fig. 5) and by tension in main flexural steel. Once the shear related crack develops, the beam transformed quickly into a tied-arch which exhibits considerable reserve capacity.

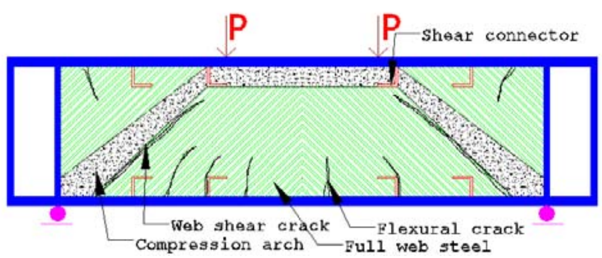

Fig. 5 Tied-arch action in a box sandwich beam

Numerical Approach

Generally in a numerical simulation of a physical process, a suitable numerical technique is adopted to evaluate a mathematical model of that physical process. No doubt finite element method(FEM) is one of the powerful numerical techniques by which any complex physical process can be simulated precisely. In this study none of the above analogies is directly applied, rather macro physical model for shear resisting mechanism will be presented based on a series of numerical experimentation with FEM simulation.

Experimental Program

Experimental study was conducted as a reference or guideline for the analytical procedure. Most of the analytical specimen was kept similar to that of the experimental one to ensure that analytical result is going on the same line of experimental one. On this goal a good number of both box and open type sandwich beams specimens were tested as shown in Table 1. In this Table expBN1 to expBN5 are box sandwich and expH327 to expL457 are open sandwich specimens. The experimental setup was in accordance to Fig. 6.

Table. 1 Experimental specimens

Specimen	f'_c MPa	Full web f_{wy}	Pw %	Failure mode test	FEM	Shear strength Test kN	FEM kN
expBN1	24.4	-	0	SC[1]	SC	291	248
expBN2	26.0	319	1.20	SC	SC	603	576
expBN3	24.7	324	1.80	SC	SC	722	720
expBN4	24.4	342	2.40	SC	SC	796	759
expBN5	22.3	342	1.20	SC	SC	554	483
expH327	36.0	250	2.13	SC	SC	330	329
expH457	34.3	249	3.00	SC	SC	381	369
expH607	33.5	249	4.00	SC	SC	426	379
expH453	33.9	249	3.00	SC	SC	344	310
expL457	36.9	249	3.00	SC	SC	337	320

1-SC: shear compression failure

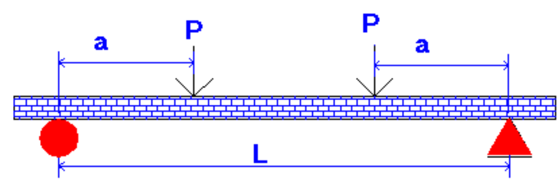

Fig. 6 4-point Loading condition of the test specimen.

Analytical Program

The aim of this study is to develop rational shear strength equations with a broad spectrum or bandwidth. On this regard a series of beams were analyzed with different variable parameters. They are as follows.

- Shear span to depth ratio (a/d)
- Compressive strength of concrete (f'_c)
- Percent of web steel (P_w)
- Percent of Flexural Steel (P_s)
- Yield strength of web steel (f_{wy})

Shear Resisting Model

The shear resisting model is divided into two parts

- Concrete model
- Steel model

Concrete Model

Types of cracks in a sandwich beam are very similar to those in a usual RC beam (Fig.7) as shown in Fig.8. But it is quite uncertain to know the exact location of crack which cause

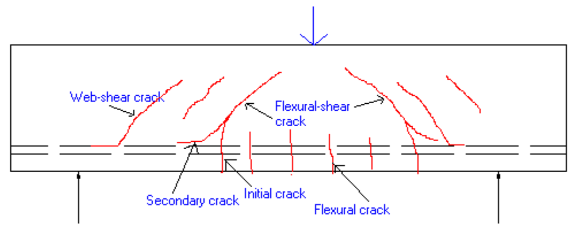

Fig. 7 Type of crack in RC beam

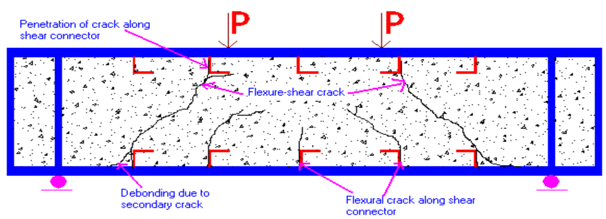

Fig. 8 Formation of crack in box sandwich beam

the beam to fail along the crack path. So in this model (Fig. 9) the following assumptions were considered to locate the critical or main crack.

- The crack with a minimum depth of compression zone should be a critical one. Because this crack has a maximum depth of penetration into the compression zone. And this compression zone experience maximum shear stress.
- Near the neutral axis the critical crack should have an inclination angle of 45°.
- A circular arc with a 45° chord inclination from the beneath of compression zone to the extreme bottom fiber of the concrete is the cracking path of the critical crack.
- For open sandwich beam, in no case the crack can penetrate beyond the top flange.
- If two cracks have the same configuration then the one that have the internal shear resistance is closer to external force should be taken as critical.
- Throughout the loading history, a particular crack at a particular location will not necessarily remain same due to internal stress redistribution and rearrangement

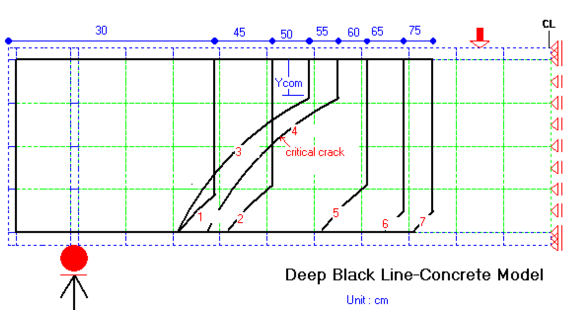

Fig. 9 Crack configuration in a box sandwich beam.

Steel Model

Compare to the concrete model, steel model is rather simple. Upon selection the critical crack, the steel model traces the same path of concrete but not in a curvilinear way. It is assumed that failure always intercept the shortest distance with the minimum area to produce maximum stress. Since a straight line is the shortest distance between two points, the steel model is furnished by joining corresponding points with straight lines as shown in Fig.10

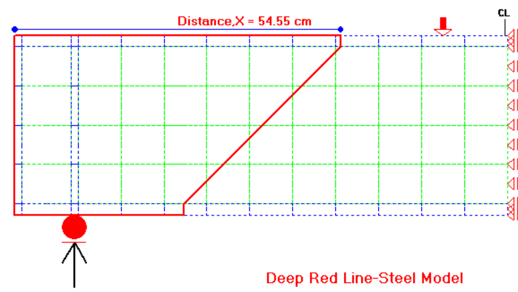

Fig. 10 Steel model along the critical crack path

Shear Strength Equations

A set of empirical equations is developed considering the corresponding affecting parameter(s). The stresses are,

- Shear stress in compression zone, S_{cz} (Eq.1)
- Interlocking stress at cracked zone, S_i (Eq.2)
- Shear stress at top flange, S_{tf} (Eq.3)
- Shear stress at web steel , S_{web} (Eq.4)
- Shear stress at bottom flange, S_{bf} (Eq.5)

Here all the stresses are in MPa
The empirical equations are,

$$S_{cz} = 0.54 f'_c - 2.53\frac{a}{d} \quad \text{and} \quad \geq 1 \qquad \text{.........(1)}$$

$$S_i = 0.32 P_s + 0.132 f'_c - 0.55\frac{a}{d} \quad \text{and} \geq 1.5 \quad \text{.....(2)}$$

$$S_{tf} = 2.22 P_s - 5.5 P_w + 1.38 f'_c - 10\frac{a}{d} \quad \text{and} \geq 0.5 \qquad \text{..............(3)}$$

$$S_{web} = 8.62 P_s + 8 P_w + 0.21 f_{wy} - 16.5\frac{a}{d} \qquad \text{......(4)}$$

$$S_{bf} = 0.44 P_s + 0.53 f'_c \qquad \text{...............(5)}$$

Here,
f'_c = compressive strength of concrete (Mpa)
f_{wy} = yield strength of web steel (MPa)
Pw = percent of web steel (%)
Ps = percent of flexural steel (%)
Es= young's modulus of steel (GPa)
Depth of compression zone, Y_{com} is calculated by Eq.6.

$$Y_{com} = d(0.011 P_s + 0.0092 f'_c - 0.0242\frac{a}{d}) \qquad \text{.....(6)}$$

Depth of cracked concrete is

$Y_{ck} = h_{web} - Y_{com}$

h_{web} = hight of the web steel

$= h - h_{tf} - h_{bf}$: h_{tf} = heigth of the top flange

: h_{bf} = height of the bottom flange

The shear strength is then calculated by the following equations.

Shear in concrete, V_c is

$$V_c = V_{cz} + V_{iy}$$
$$= S_{cz} \times A_{cz} + S_i \times A_i \times Sin\theta$$
$$= S_{cz} \times y_{com} \times b + S_i \times l_i \times b \times Sin\theta$$

Shear in steel, Vs is

$$V_s = V_{tf} + V_{web} + V_{bf}$$
$$= S_{tf}\ \mathrm{A}_{tf} + S_{web} A_{web} \cos\theta + S_{bf} A_{bf}$$
$$= S_{tf} \times b_{tf} \times h_{tf} + S_{web} \times t_{web} \times h_{web} + S_{bf} \times b \times h_{bf}$$

total shear $V = Vc + Vs$

Crack Configuration

To know the exact geometry of the crack, additional parameters are needed as shown in Fig. 11. They are,

- inclination at the top of the crack *(θ_t)*
- inclination at the bottom of the crack *(θ_b)*
- radius of curvature of the crack *(R)*
- center of curvature of the crack *(C_x,C_y)*
- distance from loading point *(X_{load})*

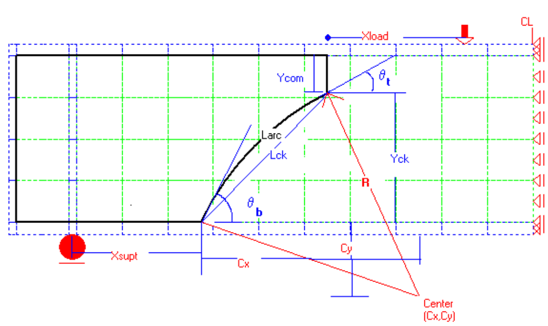

Fig.11 Geometric parameters of crack

$$\theta_b = 18.36\frac{a}{d} + 27 \quad \text{and} \quad \leq 90^0 \qquad ....(7)$$

$$\theta_t = 17.5\frac{a}{d} + 20 \quad \text{and} \quad \leq 75^0 \qquad ....(8)$$

$$X_{load} = a \times 0.41 e^{-0.38 Pw} \qquad ....(9)$$

After knowing θ_t and θ_b, the radius of curvature of crack can be calculated by

$$R = 1.11 \frac{Y_{ck}}{\sin 45^0} \times \frac{180}{\Pi(\theta_b \approx \theta_t)} \qquad ....(10)$$

Here,

arc length/chord length = Larc/Lck = 1.11

The center of curvature of crack is given by

$$C_x = R \sin\theta_b \qquad ....(11)$$

$$C_y = R \cos\theta_b \qquad ....(12)$$

Discussion

Shear strength from analysis and from above mentioned equations shows a good agreement between the two. The model, which was then verified for a series of beams, changing all of their affection parameters, is applicable for a large proportion of beams unless some parameters go beyond usual boundary. Fig.12 shows that all results fall within 90% bandwidth of each other.

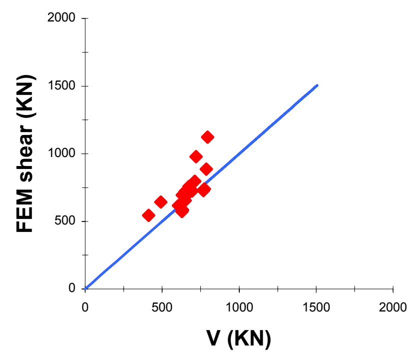

Fig.12 Comparison of shear strength

Conclusion

A physical model, which contains of a set of formulae, for prediction of shear strength of box sandwich members was presented based on the numerical experiment with nonlinear FEM analysis. The model contains the concrete part and the steel part. The former represents the contribution of flexural compression zone and diagonal cracking zone, while the latter represents the contribution of web plate and lower/upper flange plates. The results predicted by the model agree well with the FEM results.

At present the results of the FEM rather underestimate the experimental results. After improving this discrepancy the above model can be improved. Furthermore this model can be expanded to predict the shear strength of open sandwich members

(Note: Same procedure is followed for open sandwich beam as well)